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A METHOD OF FINDING THE EVOLUTE OF THE FOUR-CUSPED HYPOCYCLOID.

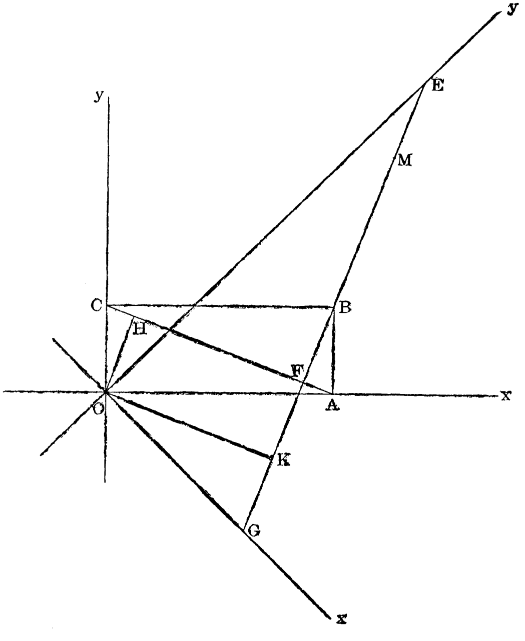
By Prof. R. H. Graves, Chapel Hill, N. C.

The following is a method, based on the Theory of Roulettes, of proving the well-known result, that the evolute of a four-cusped hypocycloid is also a four-cusped hypocycloid. Let $AC = a$ move with its extremities on the rectangular axes Ox and Oy. The envelope of AC is the four-cusped hypocycloid.

Complete the rectangle $OABC$; draw Ox' and Oy' bisecting the angles between Ox and Oy; draw $EBFG$ perpendicular to AC; draw OK and OH parallel and perpendicular to AC; let $CAO = \theta$.

B is the instantaneous centre, BF is the normal to the envelope of AC, and its envelope is the required evolute.

$$ABF = \theta,$$
$$FBC = 90^\circ - \theta,$$
$$BEO = 45^\circ - \theta,$$
$$OGK = 45^\circ + \theta.$$



$$OK = HF = a - 2a \sin^2 \theta = a \cos 2\theta.$$

$$EG = OK \operatorname{cosec} OGK \operatorname{cosec} BEO = a \cos 2\theta \operatorname{cosec} (45^\circ + \theta) \operatorname{cosec} (45^\circ - \theta).$$

Hence, by reduction, $EG = 2a$.

Since EG is of constant length, and its extremities move on two rectangular axes, its envelope must be a four-cusped hypocycloid, which is the required evolute.

Remark.—If M is the point where EG touches its envelope, $BM = BK$.

For, at the point (x, y) of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$, the radius of curvature is $3\,(axy)^{\frac{1}{3}}$; and the perpendicular from the origin on the tangent is $(axy)^{\frac{1}{3}}$.

Or, it follows from the formula connecting the segments into which the radius of curvature of the hypocycloid is divided by the instantaneous centre. (See Williamson's Differential Calculus, Art. 281.)